CHIRAL PERTURBATION THEORY AND THE $1/N_c$-EXPANSION

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We briefly review the effective theory that describes the low energy properties of QCD with three light quarks and a large number of colours, N_c, and then discuss the mechanisms that forbid the Kaplan-Manohar transformation in this framework.

1 Expansion in the quark masses and $1/N_c$

It is generally accepted[a] that QCD possesses an exact $\mathrm{U}(3)_R\times\mathrm{U}(3)_L$ flavour symmetry in the limit $m_u = m_d = m_s = 0$, $N_c \to \infty$. We furthermore assume that (a) the axial part of this symmetry is spontaneously broken and (b) that, in the vicinity of this point, the low energy properties of the theory are governed by the Goldstone bosons associated with this symmetry breakdown. These assumptions provide the basis for the effective theory, where the nine Goldstone degrees of freedom are collected in a matrix $U(x) \in \mathrm{U}(3)$. Apart from the Wess-Zumino term, the effective Lagrangian represents the most general expression formed with the field U and the source fields for the quark currents that is consistent with chiral symmetry. Furthermore, in order to study the consequences of the $\mathrm{U}(1)_A$-anomaly, we include a source field for the winding number density $\omega = 1/(16\pi^2)\,\mathrm{tr}_c\, G_{\mu\nu}\widetilde{G}^{\mu\nu}$. We denote this field by $\theta(x)$; in the QCD-Lagrangian it enters in the form $\mathcal{L}_{\mathrm{QCD}} = -\theta\,\omega + ...$

The terms in the effective Lagrangian are ordered by introducing a counting parameter δ, $\mathcal{L}_{eff} = \mathcal{L}_1 + \mathcal{L}_\delta + \mathcal{L}_{\delta^2} + \ldots$ A coherent expansion emerges if powers of momenta, quark masses and $1/N_c$ are counted according to [2]

$$p = O(\sqrt{\delta})\ , \quad m = O(\delta)\ , \quad 1/N_c = O(\delta)\ . \tag{1}$$

The effective coupling constants are proportional to powers of N_c. To determine these, one infers the known large N_c behaviour of the QCD correlation functions [3]. These state in particular that correlation functions involving the winding number density n times are suppressed by a factor of N_c^{-n}. As shown in ref. [1], this implies that the dependence of the effective Lagrangian on the field θ is polynomial, to every order in the expansion in powers of δ.

[a] See ref. [1] for a detailed list of references

2 The Kaplan-Manohar transformation

Kaplan and Manohar [4] pointed out the existence of an ambiguity in the quark mass ratios inherent to the three flavour effective Lagrangian at fixed N_c. A priori, this situation persists also if the number of colours is taken large: In the effective Lagrangian, one may replace the quark mass matrix m with

$$m' = m + \lambda\, e^{-i\theta} m^{\dagger -1} \det m^\dagger \;, \tag{2}$$

without violating chiral symmetry. Under a chiral transformation, the mass matrix transforms according to $m \rightarrow V_R m V_L{}^\dagger$, with $V_R, V_L \in \mathrm{U}(3)$. The matrix m' transforms in the same manner since the factor $e^{-i\theta}$ transforms contragrediently to $\det m^\dagger$ under $\mathrm{U}(1)_A$-rotations. The Lagrangian $\mathcal{L}_{\mathit{eff}}(m')$ is, however, in conflict with large N_c, because, in view of the factor $\lambda\, e^{-i\theta}$, it involves arbitrary powers of the field θ. As stated in the preceding section, the original Lagrangian, $\mathcal{L}_{\mathit{eff}}(m)$, depends on this variable only polynomially. The conflict is resolved only if the parameter λ vanishes to all orders in $1/N_c$. [1]

There is no reason why the effective theory should allow a transformation of the type (2) in the first place – after all QCD possesses no corresponding symmetry. It is for instance well-known that correlation functions of the scalar or pseudoscalar currents are not invariant under a change of the mass matrix. Experimental information on these would pin down the values of the quark mass ratios also in the standard framework [5]. The generalized version of the Kaplan-Manohar transformation in eq. (2) necessarily involves the vacuum angle θ, as a consequence the correlation functions of the winding number density ω also fail to be invariant. We do not have experimental information on these either, but they are constrained theoretically at large N_c. As it happens, the transformation in eq. (2) violates these constraints.

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